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## Développements récents dans l'analyse cepstrale appliquée à la mécanique

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## SCHÉMA DE LA PRÉSENTATION

- **Première histoire, le contexte et les définitions**
- **Nouvelle méthode pour éditer les signaux de temps en utilisant le cepstre réel**
- **Applications dans le diagnostic des machines**
  - détection et modification des modèles d'harmoniques et des bandes latérales: engrenages, roulements et turbomachines
  - Séparation des fonctions d'excitation et de transfert
  - Compensation des variations de vitesse - différente pour les engrenages et les roulements
- **Applications en analyse modale opérationnelle**
  - Pré-traitement pour éliminer les fonctions d'excitation déterministes
  - Extension à la vitesse variable

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## PAPIER ORIGINAL SUR LE CEPSTRE

Bogert, Healy and Tukey (1963) "*The Quefreny Alanysis of Time Series For Echoes; Cepstrum, Pseudo-autcovariance, Cross-cepstrum and Saphe Cracking*". *Proc. Symp. On Time Series Analysis, Wiley.*

Le cepstre defini comme "le spectre de puissance du logarithme du spectre de puissance" – utilisé pour detecter le temps de delai d'un echo dans un signal seismique (mieux que la fonction d'autocorrélation)

**Nouveau langage du Cepstre (en anglais)**

SPECtrum	→	CEPStrum
FREQUency	→	QUEFRency
HARmonic	→	RAHmonic
FILter	→	LIFter
MAGnitude	→	GAMnitude
PHASe	→	SAPHe
RADius	→	DARIus
DEMODulation	→	DEDOMulation

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## Contexte de la définition originale

Co-auteur Tukey était également l'un des deux auteurs de l'algorithme de FFT, mais deux ans plus tard (1965). La définition originale du cepstre comme le «spectre de puissance du logarithme d'un spectre de puissance» vient apparemment du fait que logiciel simple n'était pas facilement disponible pour la transformation de Fourier complexe, même à peine deux ans avant la publication de la FFT. Cela signifie que «lifrage» a été réalisé par filtrage convolutif du spectre logarithmique, plutôt que par fenêtrage dans le cepstre

**Extrait de l'article original:**

"Bien que les techniques spectrales, impliquant des opérations du deuxième degré, sont maintenant tout à fait familier, les techniques de Fourier au premier degré, apparemment simples, sont moins bien connues". Ce fut apparemment écrit par Tukey lui-même, comme référence est immédiatement faite à un papier dont il est seul auteur

Peu de temps après la publication de la FFT, le cepstre a été redéfini comme «transformée de Fourier inverse du logarithme du spectre de puissance" qui était réversible au spectre de puissance après "lifrage" dans le cepstre

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## Early work at Bell Labs and MIT

- Noll (1967) used power cepstrum to detect voiced speech and measure voice pitch
- AV Oppenheim (1964) defined “homomorphic filtering” where non-linear systems converted to linear

eg multiplication  $\xrightarrow{\text{log}}$  sum

convolution  $\xrightarrow{\text{FFT}}$  multiplication  $\xrightarrow{\text{log}}$  sum

convolution  $\xrightarrow{\text{FFT}}$  multiplication  $\xrightarrow{\text{log}}$  sum  $\xrightarrow{\text{IFFT}}$  sum

- The last is the “complex cepstrum” (RW Schafer 1968)

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## CEPSTRUM DEFINITIONS

The original definition of the (power) cepstrum was:

$$C_p(\tau) = |\mathfrak{I}\{\log(F_{xx}(f))\}|^2 \quad (1)$$

where  $F_{xx}(f)$  is a power spectrum, which can be an averaged power spectrum or the amplitude squared spectrum of a single record.

The definition of the complex cepstrum is:

$$C_c(\tau) = \mathfrak{I}^{-1}\{\log(F(f))\} = \mathfrak{I}^{-1}\{\ln(A(f)) + j\phi(f)\} \quad (2)$$

where  $F(f) = \mathfrak{I}\{f(t)\} = A(f)e^{j\phi(f)}$  (3)

in terms of the amplitude and phase of the spectrum.

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## Complex Cepstrum – Phase Unwrapping

$$C(\tau) = \mathfrak{F}^{-1}[\log(X(f))]$$

where  $X(f) = \mathfrak{F}[x(t)] = A(f)\exp(j\phi(f))$

**BUT phase must be a continuous function of frequency, ie “unwrapped”**

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## CEPSTRUM DEFINITIONS

The new power cepstrum is given by:

$$C_p(\tau) = \mathfrak{F}^{-1}\{\log(F_{xx}(f))\} \quad (4)$$

which for the spectrum of a single record (as in (3)) can be expressed as:

$$C_p(\tau) = \mathfrak{F}^{-1}\{2\ln(A(f))\} \quad (5)$$

The so-called real cepstrum is obtained by setting the phase to zero in Eq. (2):

$$C_r(\tau) = \mathfrak{F}^{-1}\{\ln(A(f))\} \quad (6)$$

which is seen to be simply a scaled version of (5).

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## APPLICATION DU CEPSTRE AU DIAGNOSTIC DES MACHINES

**A. Détection de la structure périodique dans le spectre des :**

- harmoniques (défauts dans les engrenages, les roulements, les aubes)
- bandes latérales (défauts dans les engrenages, les roulements, les aubes)
- échos, réflexions

**B. Suppression des harmoniques, bandes latérales, échos**

**C. Séparation des effets de la source et de la voie de transmission (SIMO, entrée simple, sortie multiple)**

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### Separation of Source and Transmission Path Effects (SIMO only)

$x(t)$  →  $h(t)$   
 $H(f)$  →  $y(t)$   
**Input**                      **System**                      **Output**

$$y(t) = x(t) * h(t)$$

$$Y(f) = X(f) \cdot H(f)$$

$$|Y(f)|^2 = |X(f)|^2 \cdot |H(f)|^2$$

$$\log Y = \log X + \log H$$

$$\mathfrak{T}^{-1}\{\log Y\} = \mathfrak{T}^{-1}\{\log X\} + \mathfrak{T}^{-1}\{\log H\}$$

**Thus, source and transmission path effects are additive in cepstrum. Moreover, they are often separated**

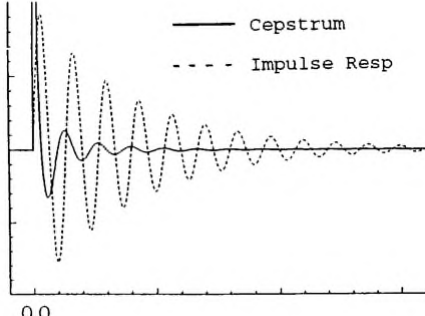
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## OPERATIONAL MODAL ANALYSIS

### Analytical Formulation of the Cepstrum (Oppenheim and Schafer, 1972) Comparison with impulse response function

**Cepstrum vs Impulse Response for an SDOF System**



**Cepstrum Equations**

$$C(n) = \ln(K) \quad , n = 0$$

$$C(n) = -\sum_i \frac{a_i^n}{n} + \sum_i \frac{c_i^n}{n} \quad , n > 0$$

$$C(n) = \sum_i \frac{b_i^{-n}}{n} - \sum_i \frac{d_i^{-n}}{n} \quad , n < 0$$

where the  $a_i$  and  $c_i$  are zeros and poles inside the unit circle and  $1/b_i$  and  $1/d_i$  are zeros and poles outside the unit circle.

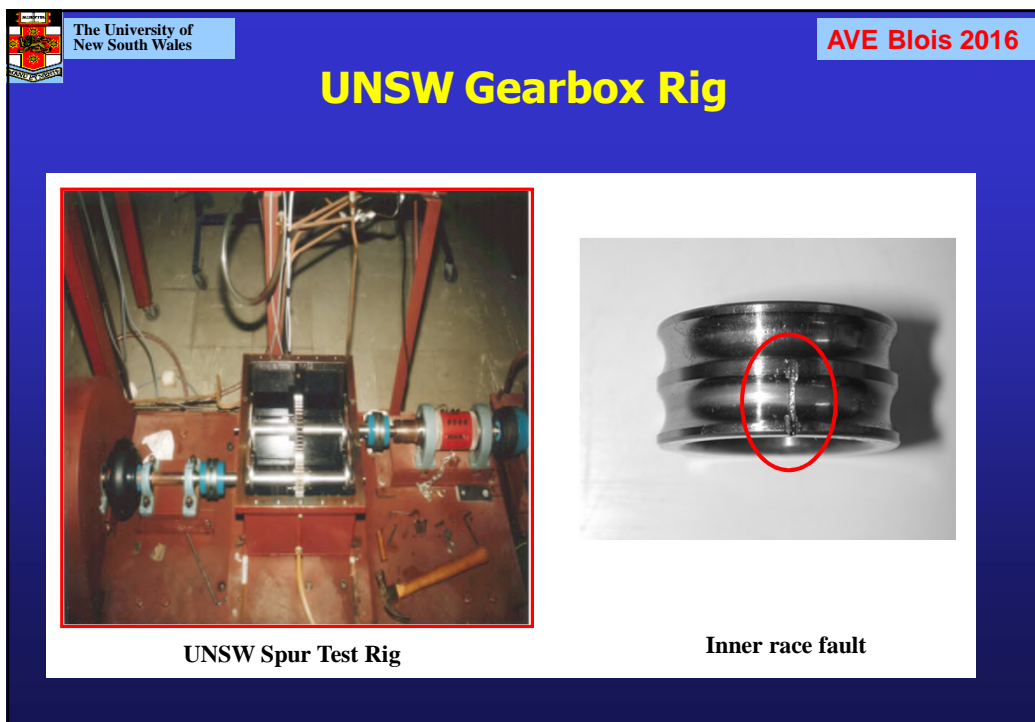
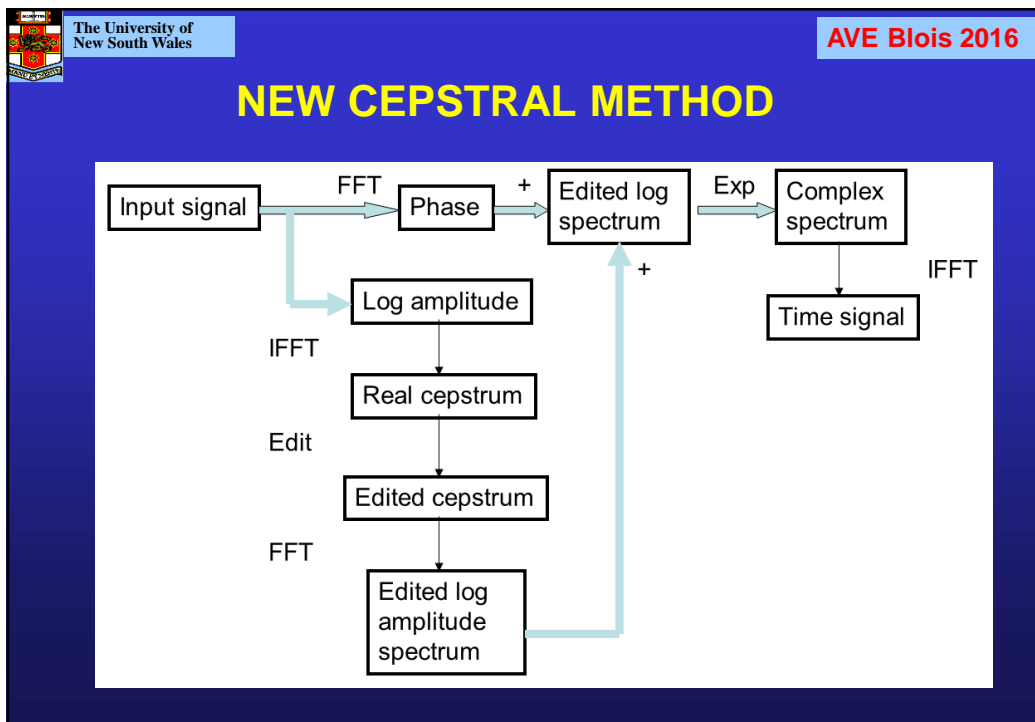
An SDOF system has one conjugate pair of poles  $c_i$  which results in an exponentially damped cosine further damped by the hyperbolic function  $1/n$

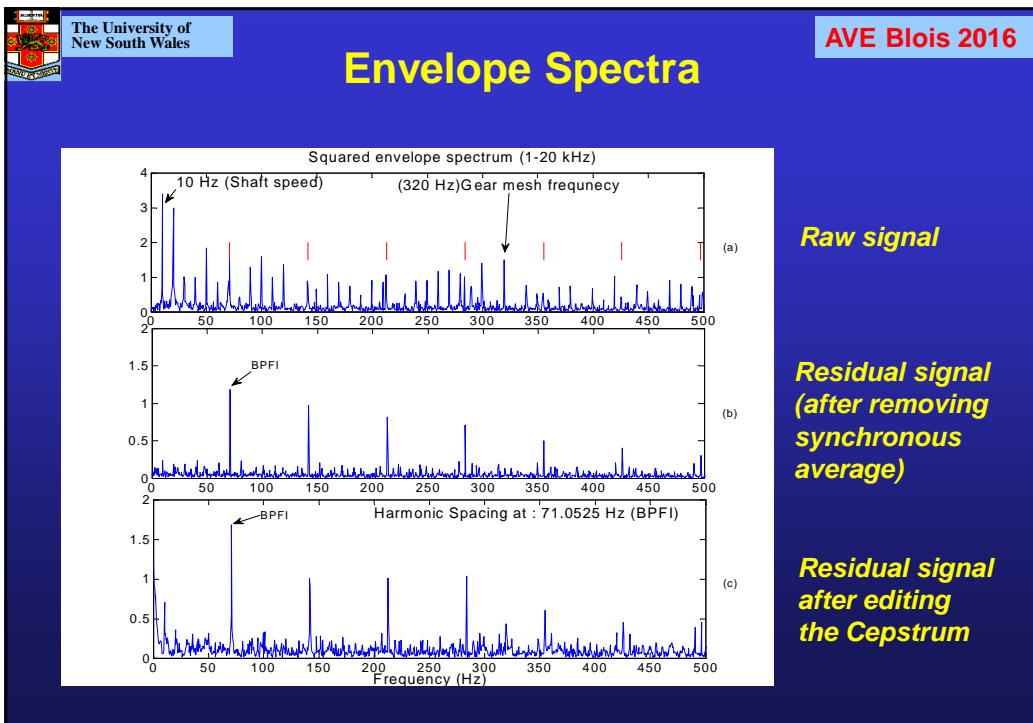
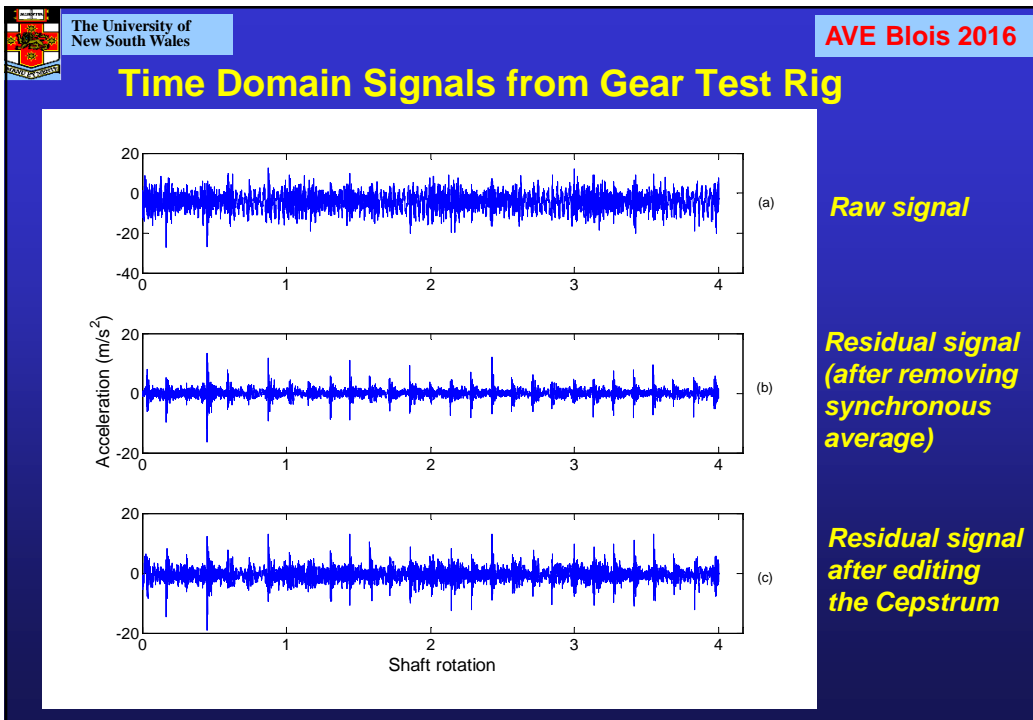
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## Nouvelle méthode pour éditer les signaux temporels

- Auparavant on pensait qu'il était nécessaire d'utiliser le cepstre complexe pour modifier les signaux de temps, par exemple suppression des échos
- Impossible à dérouler la phase des signaux d'excitation ou de réponse stationnaires, ainsi le cepstre complexe exclu
- Nouvelle méthode proposée utilise le cepstre réel pour modifier l'amplitude des signaux d'excitation ou de réponse (par exemple, supprimer les familles harmonique / bande latérale particulières, ou révéler les résonances du système) et combine le résultat avec la phase initiale pour générer des signaux de temps modifiés



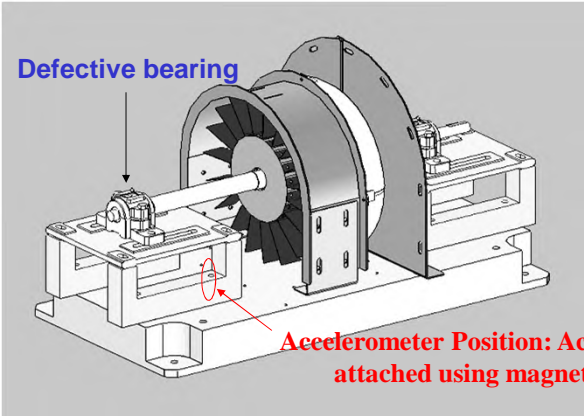




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## UNSW Fan Test rig Outer Race Fault



Defective bearing

Accelerometer Position: Accelerometer attached using magnetic base

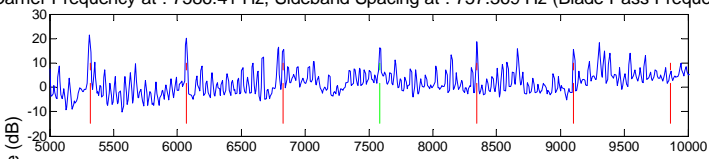


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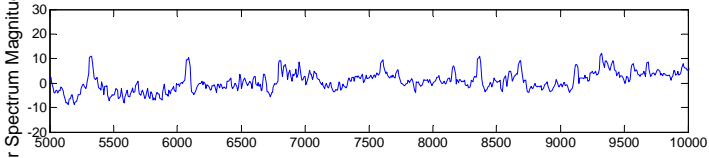
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## Power Spectra (5-10 kHz)

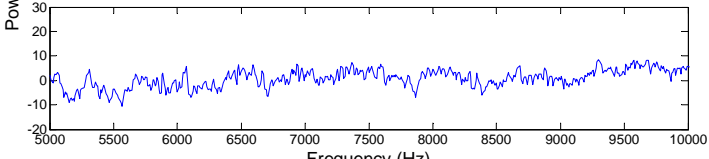
Carrier Frequency at : 7586.41 Hz, Sideband Spacing at : 757.369 Hz (Blade Pass Frequency)



Original



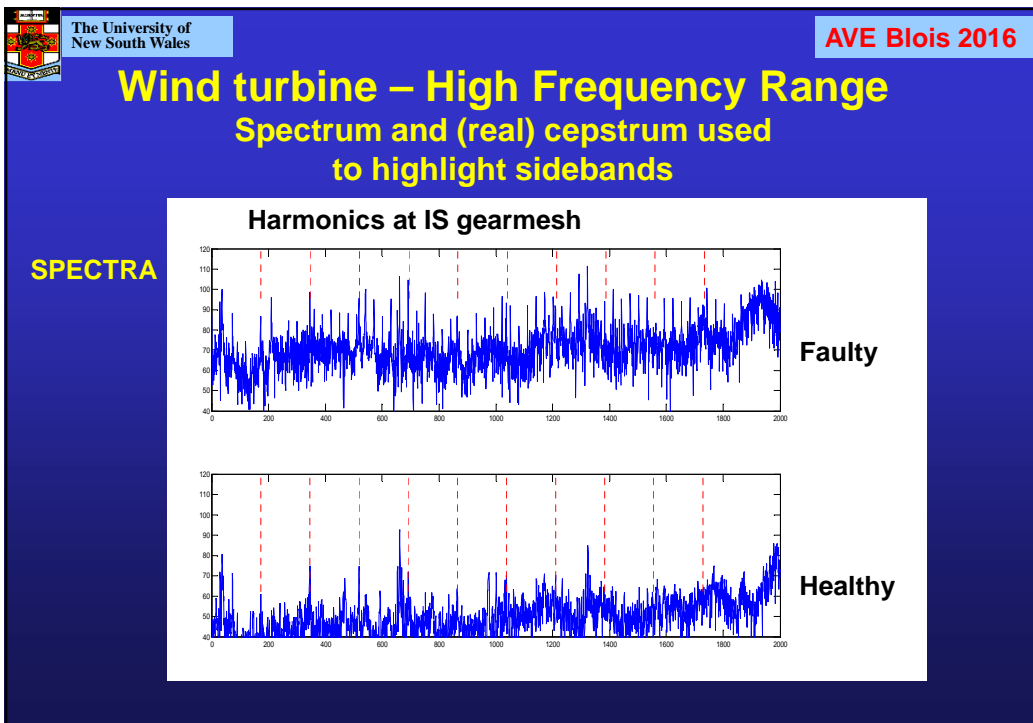
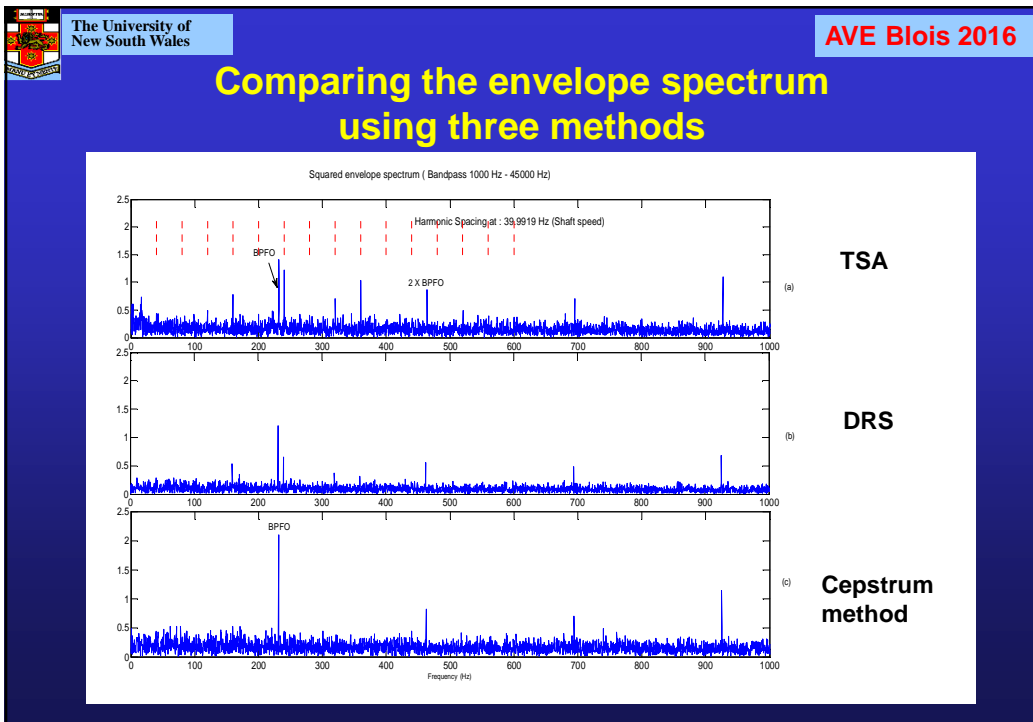
TSA

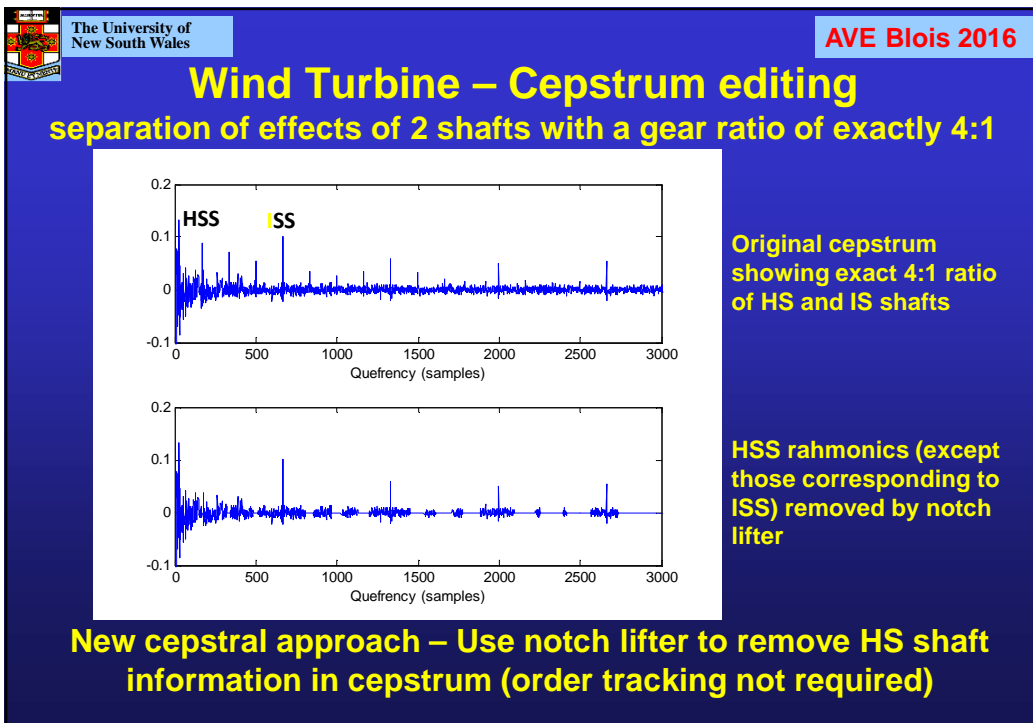
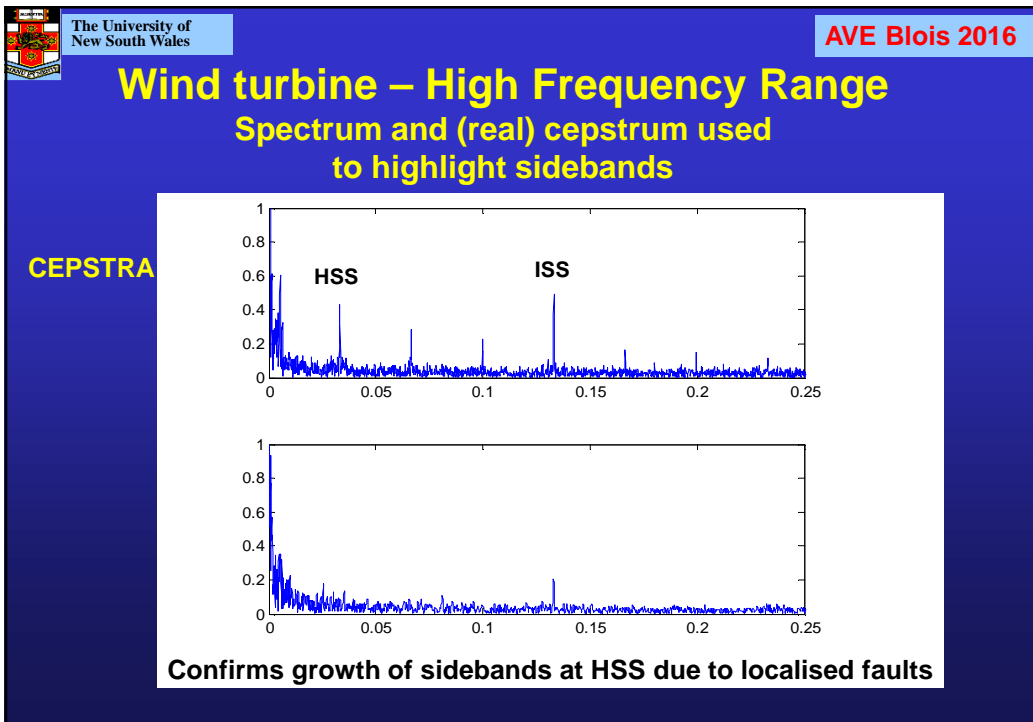


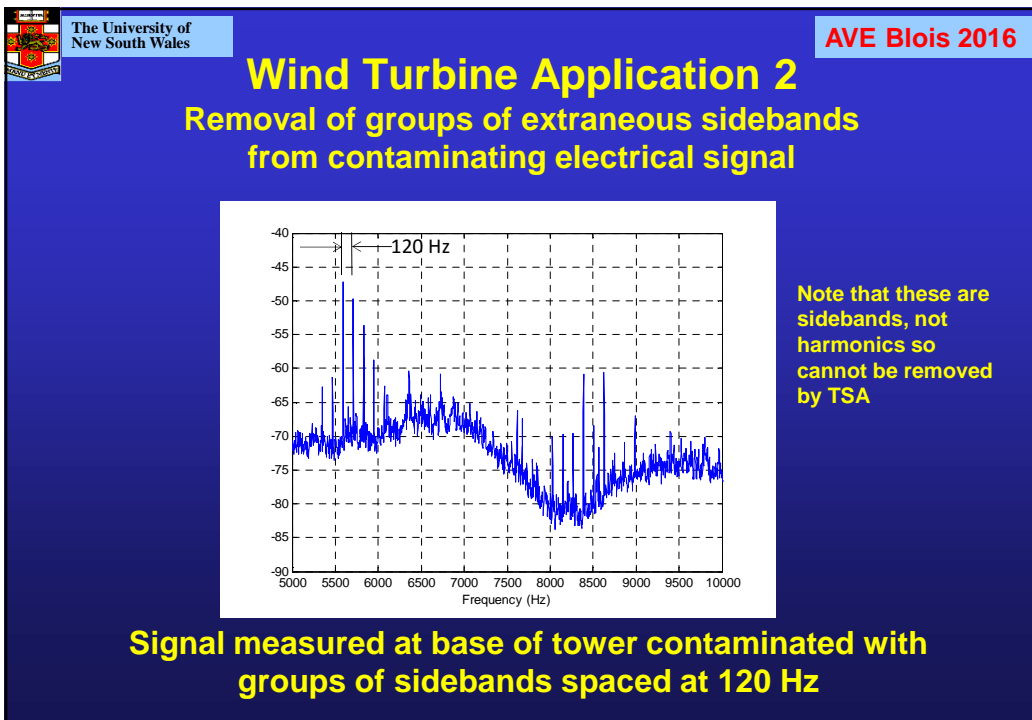
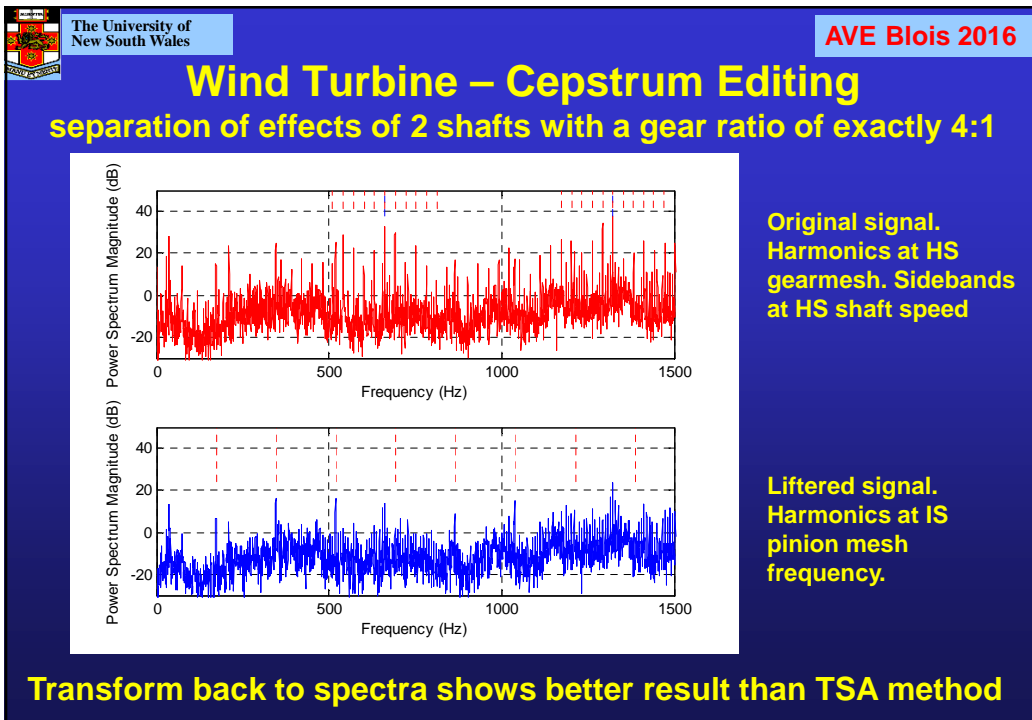
Cepstrum


Power Spectrum Magnitude (dB)

Frequency (Hz)







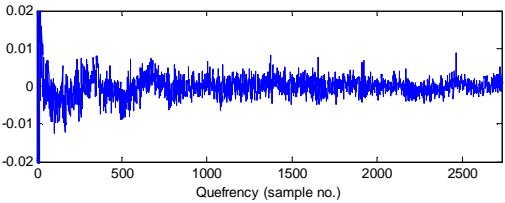


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## Wind Turbine Application 2

### Removal of groups of extraneous sidebands from contaminating electrical signal

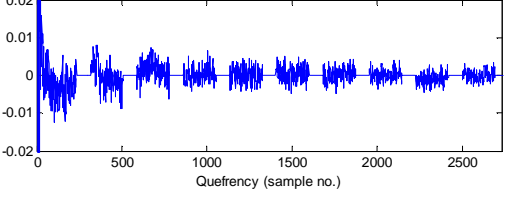


0.02  
0.01  
0  
-0.01  
-0.02

0 500 1000 1500 2000 2500

Quefrency (sample no.)

Original  
cepstrum




0.02  
0.01  
0  
-0.01  
-0.02

0 500 1000 1500 2000 2500

Quefrency (sample no.)

After liftering by  
a Type 1 lifter

Notch lifter type 1 with width  $\pm 15\%$  placed at known locations of rahmonics even if they are unclear

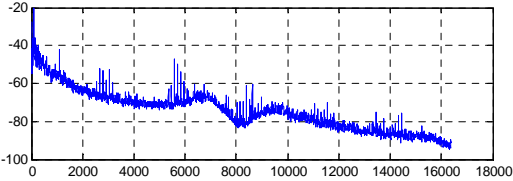


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## Wind Turbine Application 2

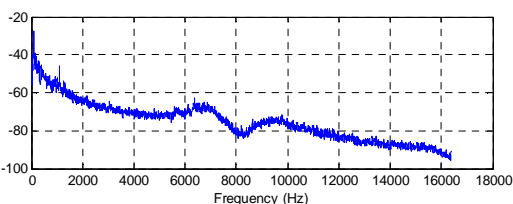
### Removal of groups of extraneous sidebands from contaminating electrical signal



-20  
-40  
-60  
-80  
-100

0 2000 4000 6000 8000 10000 12000 14000 16000 18000

Whole spectrum  
before liftering



Power Spectrum Magnitude (dB)

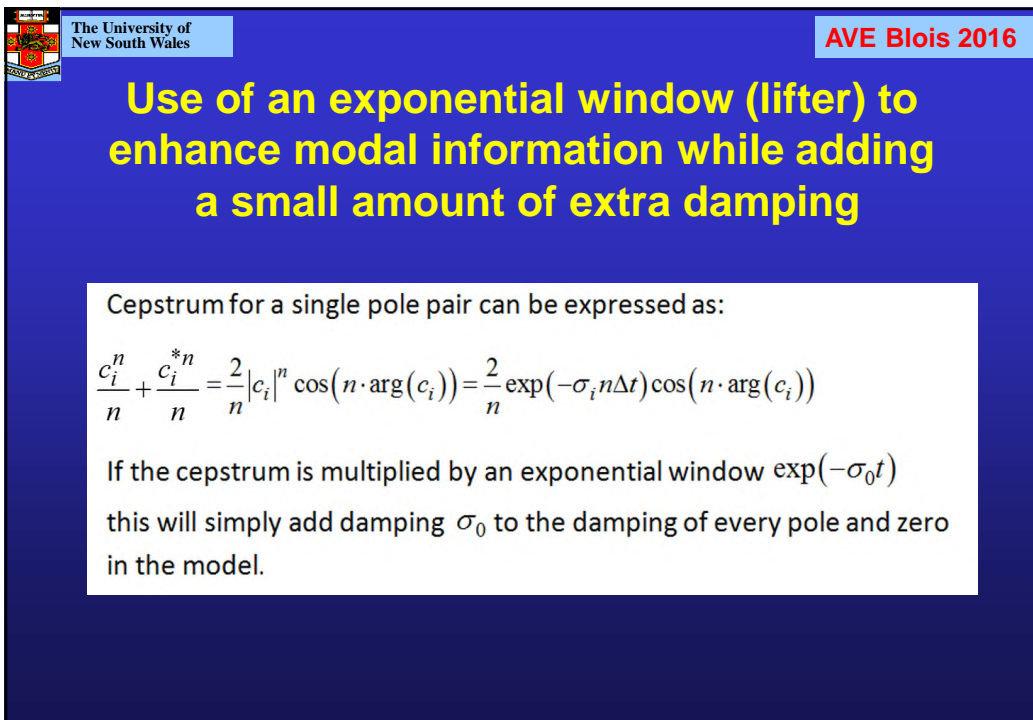
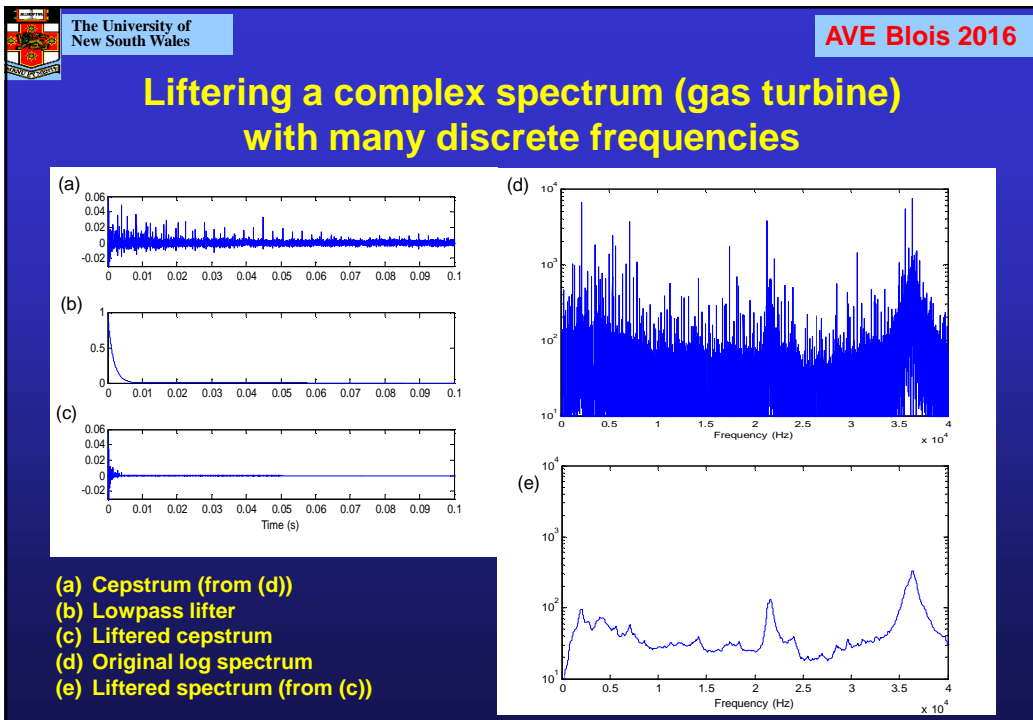
-20  
-40  
-60  
-80  
-100


0 2000 4000 6000 8000 10000 12000 14000 16000 18000

Frequency (Hz)

Whole spectrum  
after liftering

Method shown to be effective in removing the sidebands without changing rest of spectrum



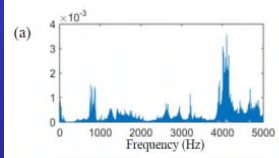


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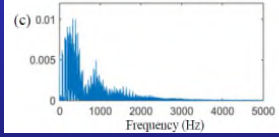
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## Gear Diagnostics – Order Tracking and Speed Determination

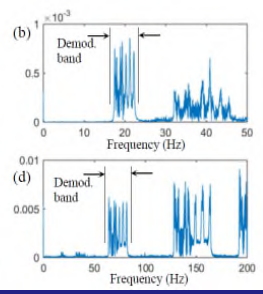
**Acceleration signal**




**Tacho signal 2 x HS shaft**



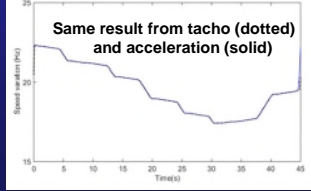
**Lowest harmonics of acceleration and tacho signals used for order tracking (phase demodulation) and speed estimation (frequency demodulation)**



**2nd harmonic of LS shaft is mixed with 1st harmonic of HS shaft, but 2nd harmonic of tacho contains same information as 1st harmonic**  
 Ratio of tacho to LS shaft is  $2 \times 46/25 = 3.68$



**Test rig – variable speed drive through speed-up gearbox (46:25) to centrifugal pump**




**Speed estimate using relationship**

$$[\omega(t)]^2 = \frac{\text{Envsq}(\dot{x}(t))}{\text{Envsq}(x(t))}$$

from frequency domain TKEO using Hilbert transform for squared envelope of the signal and its derivative

**Reference: Randall & Smith "Use of the Teager Kaiser Energy Operator to estimate machine speed", PHME conference, Bilbao, July 2016**



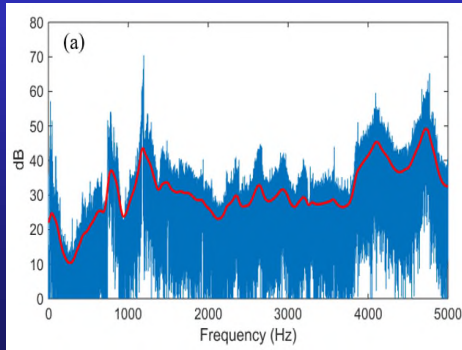
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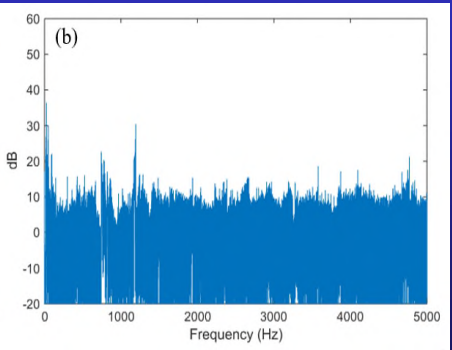
## Gear Diagnostics under Varying Speed

### Gearbox with speed variation over 45 seconds

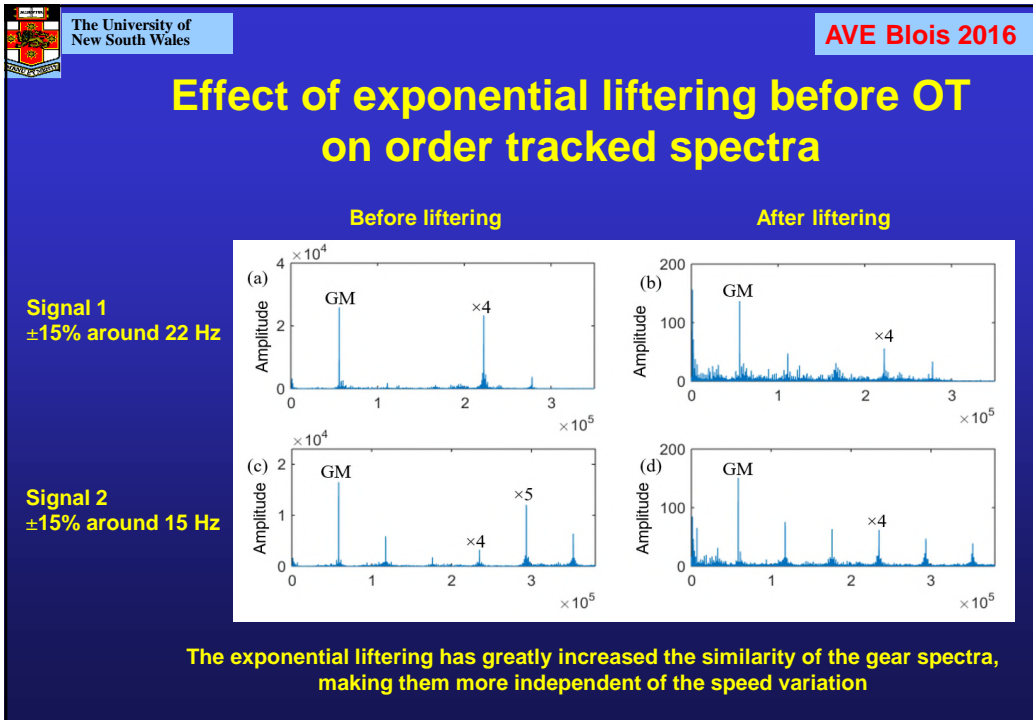
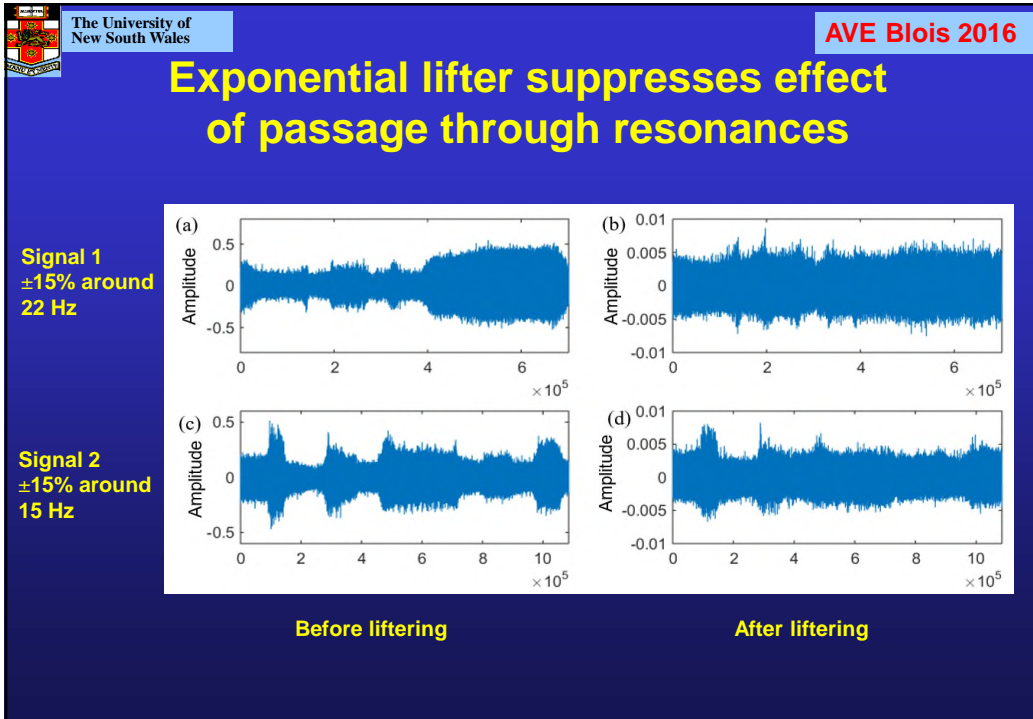
**Gear vibration signals are dominated by the forcing function rather than structural response, so now the resonance information is better removed before order tracking to reduce the amplitude modulation effects of gearmesh frequencies passing through resonances**



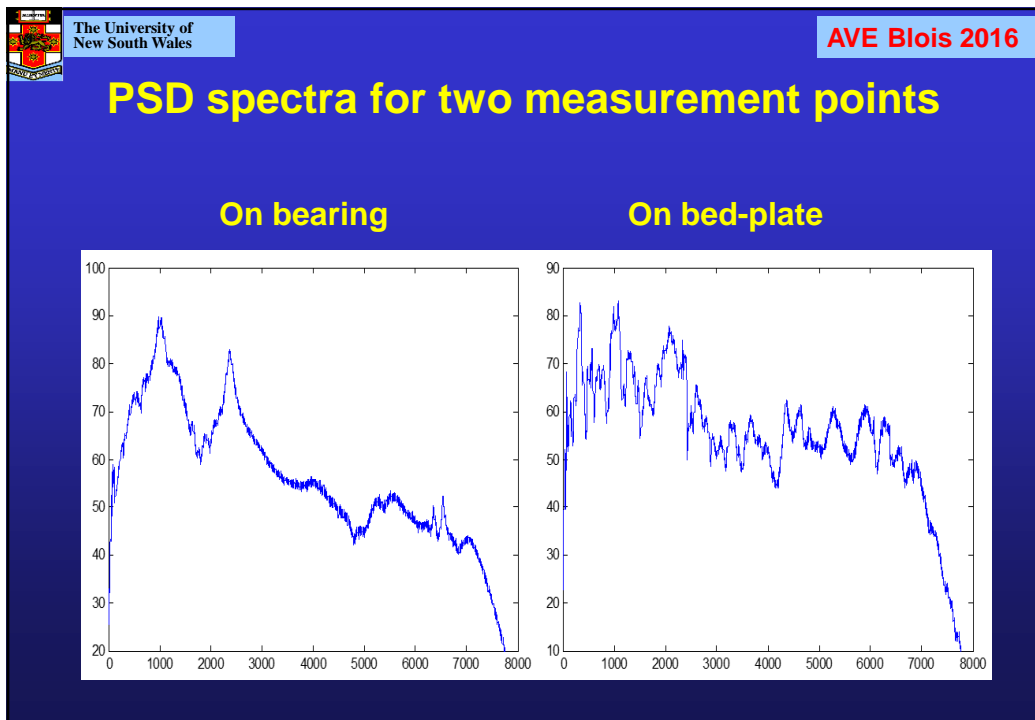
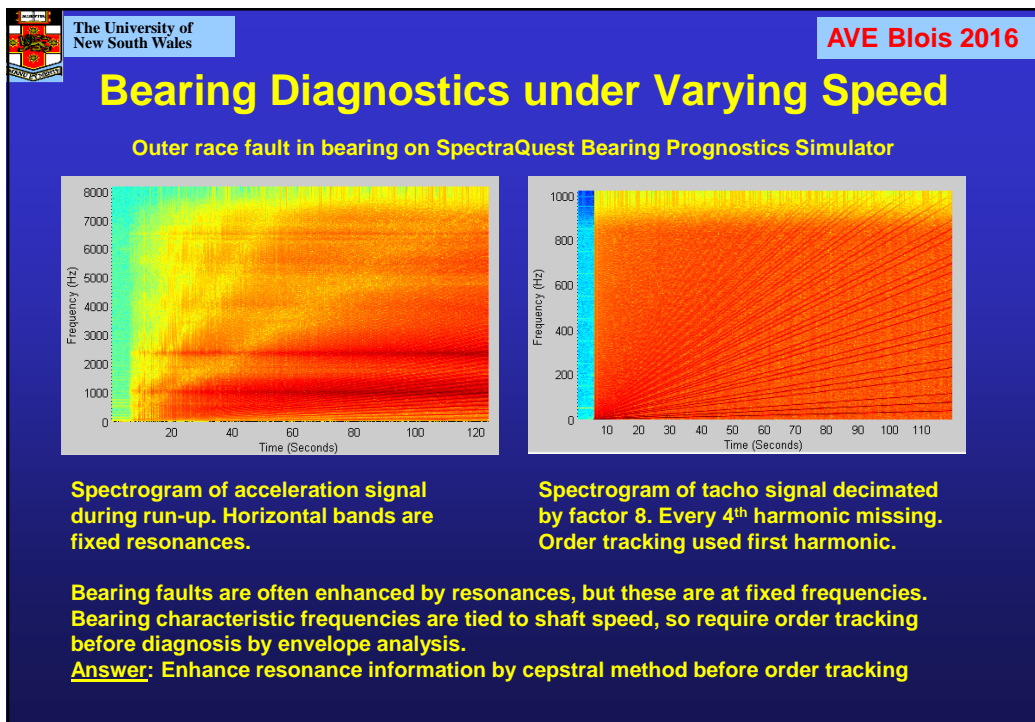
**Original spectrum and resonance spectrum generated by exponential lifter**

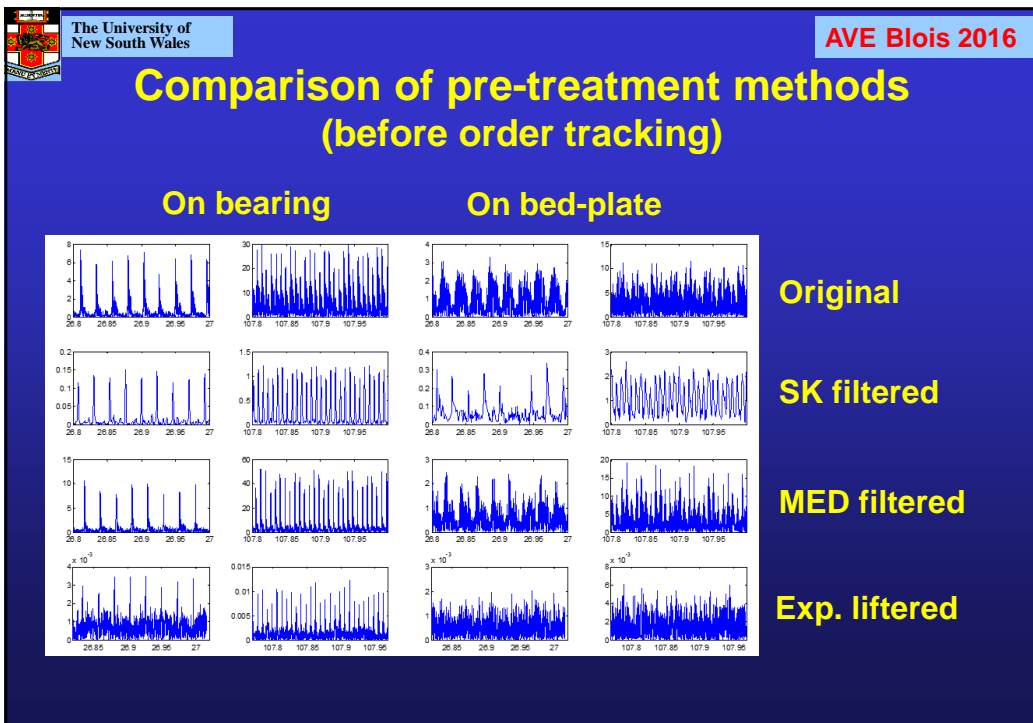
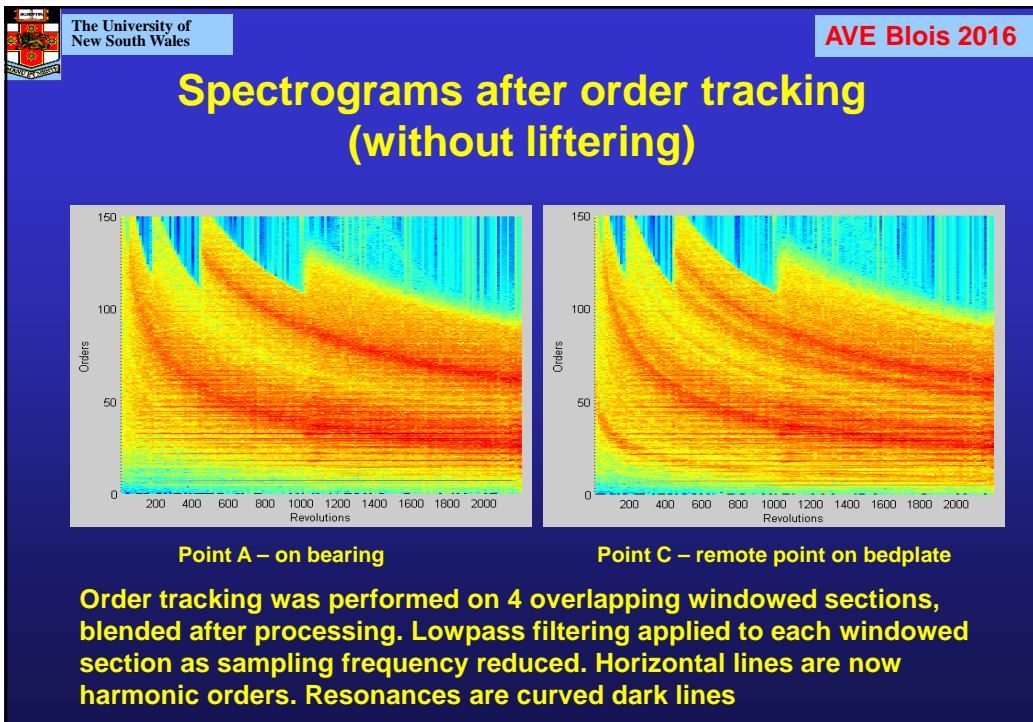


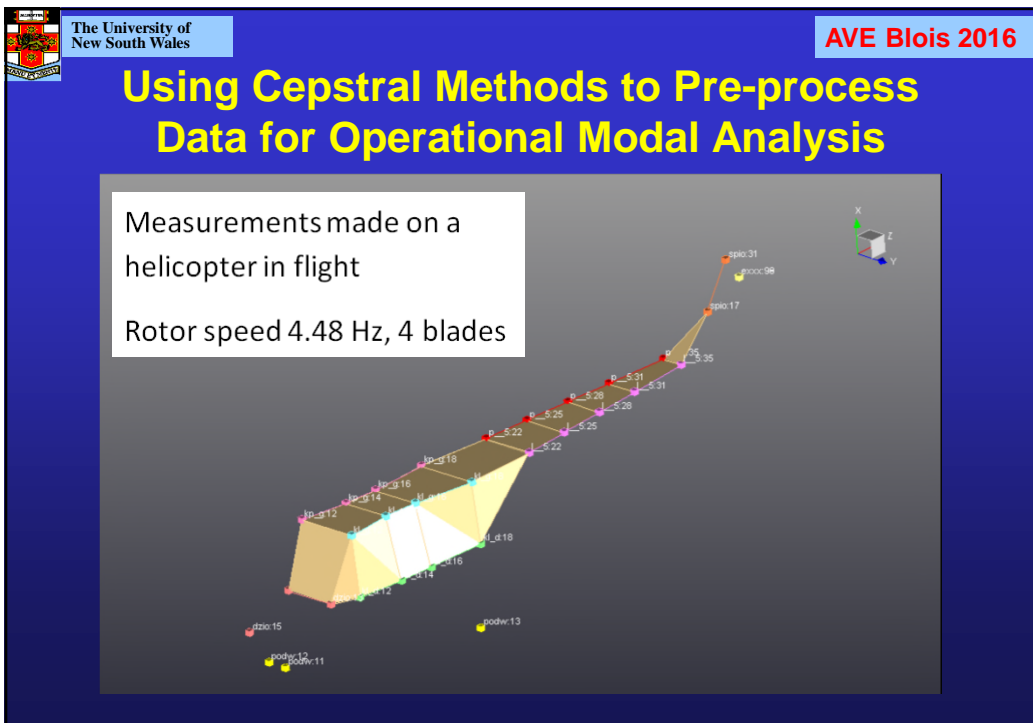
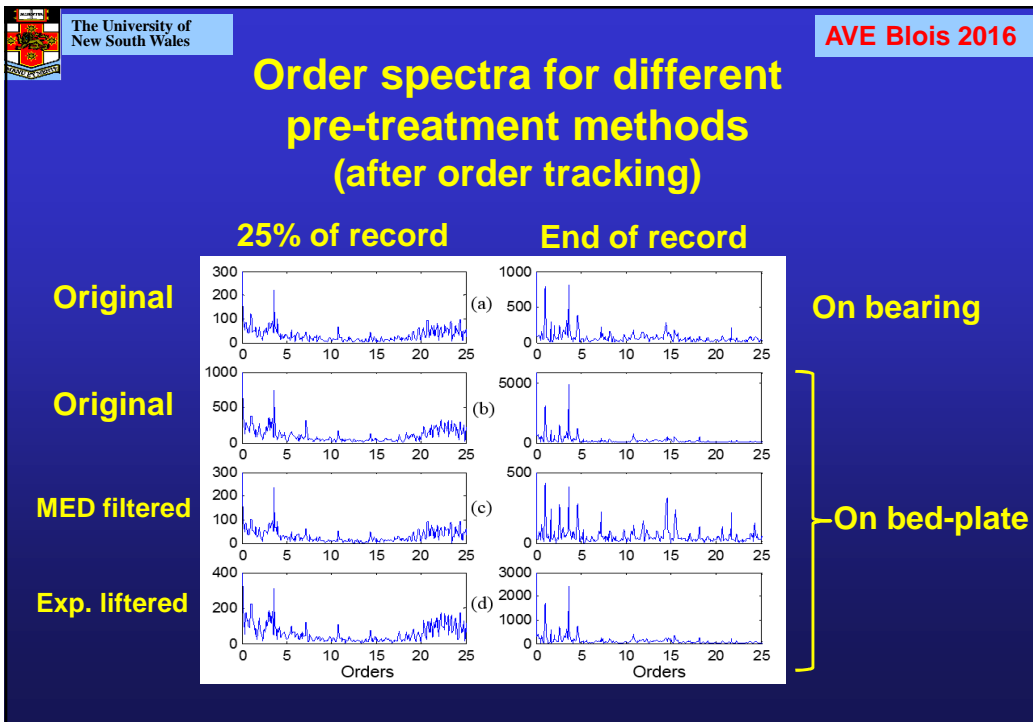
**Spectrum with resonance information removed, so dominated by forcing function**

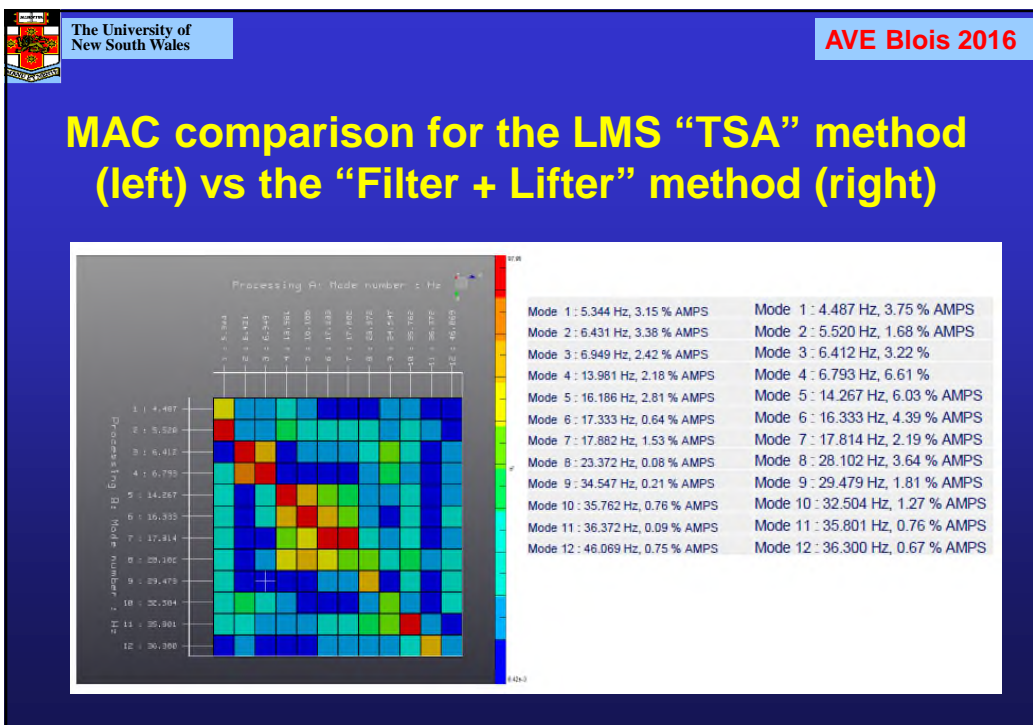
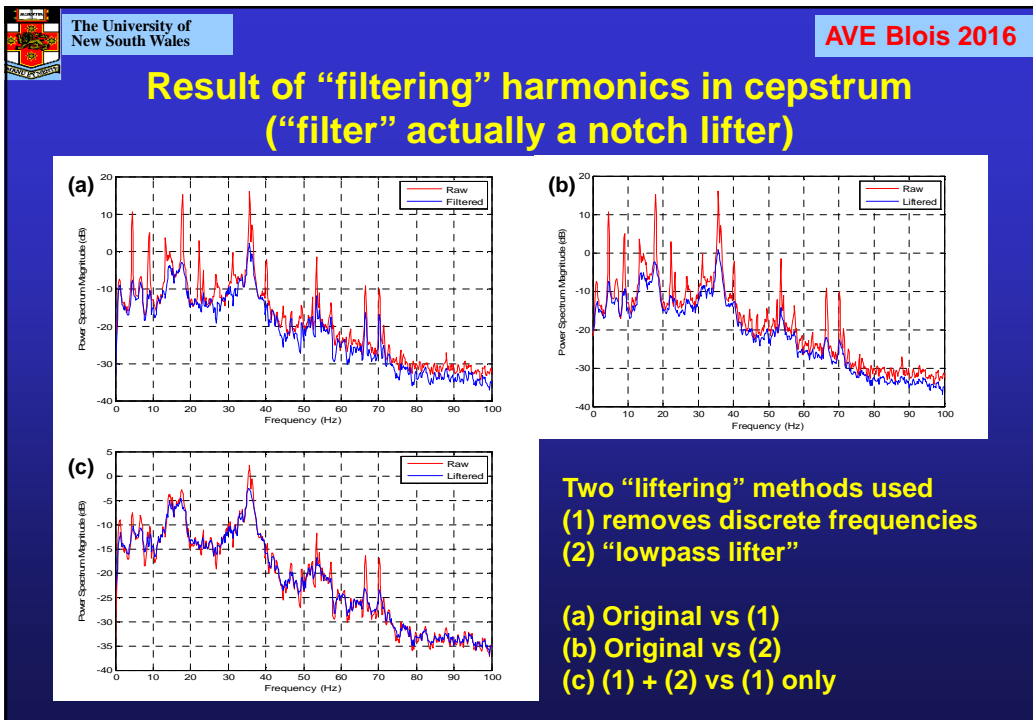












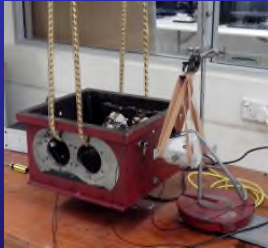


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## OMA IN THE PRESENCE OF VARIABLE SPEED HARMONIC ORDERS

Compared with the previous example of OMA on a helicopter in steady flight, if the order related components are varying in speed they can no longer be removed using TSA. Two other approaches are compared here.



Gearbox casing excited by a shaker signal with 22 orders of a "shaft speed" with a mean of 76 Hz, but varying around this by  $\pm 15\%$ , with a modulating frequency of 2 Hz (0.5s period). This was to simulate a varying speed gearbox. The intention was to carry out OMA on the casing from the response signals only, but the shaker force was measured so as to compare with EMA

Two methods were tried:

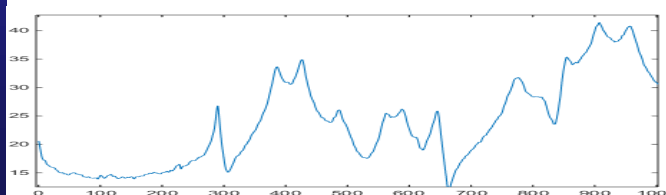
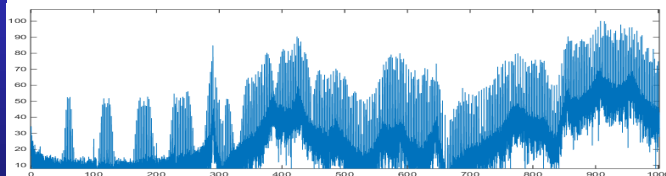
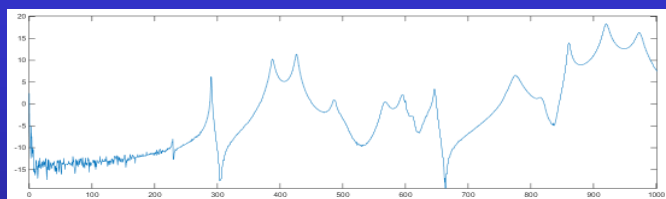
- 1) Exponential lifter on cepstrum of time signal
- 2) Transform to order domain, remove harmonics with notch lifter and transform back to time domain

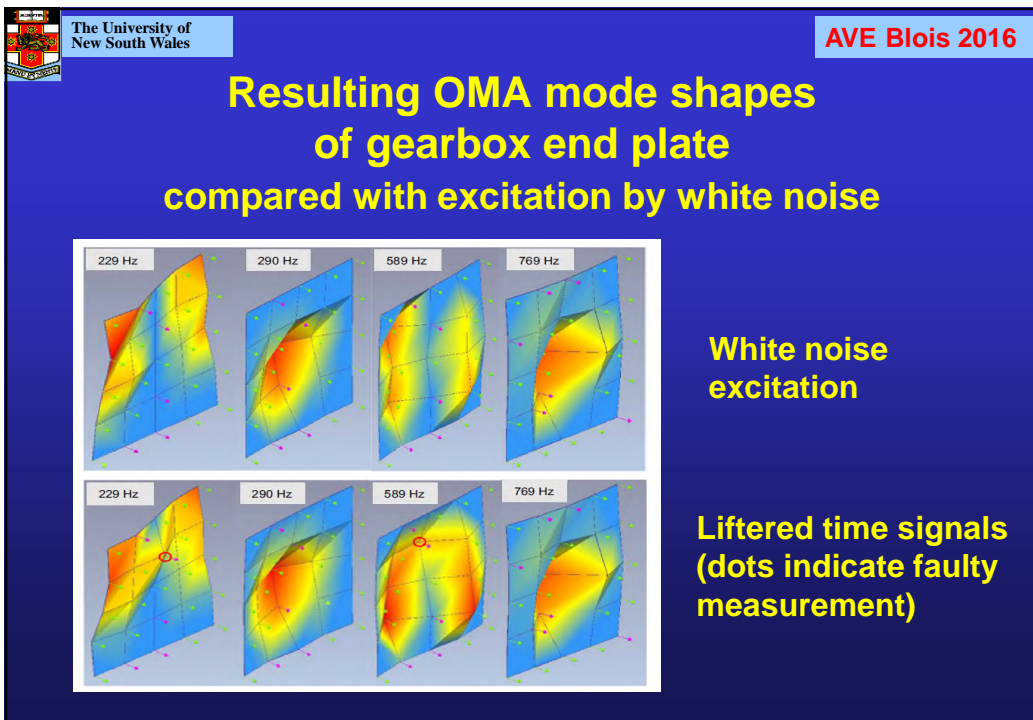
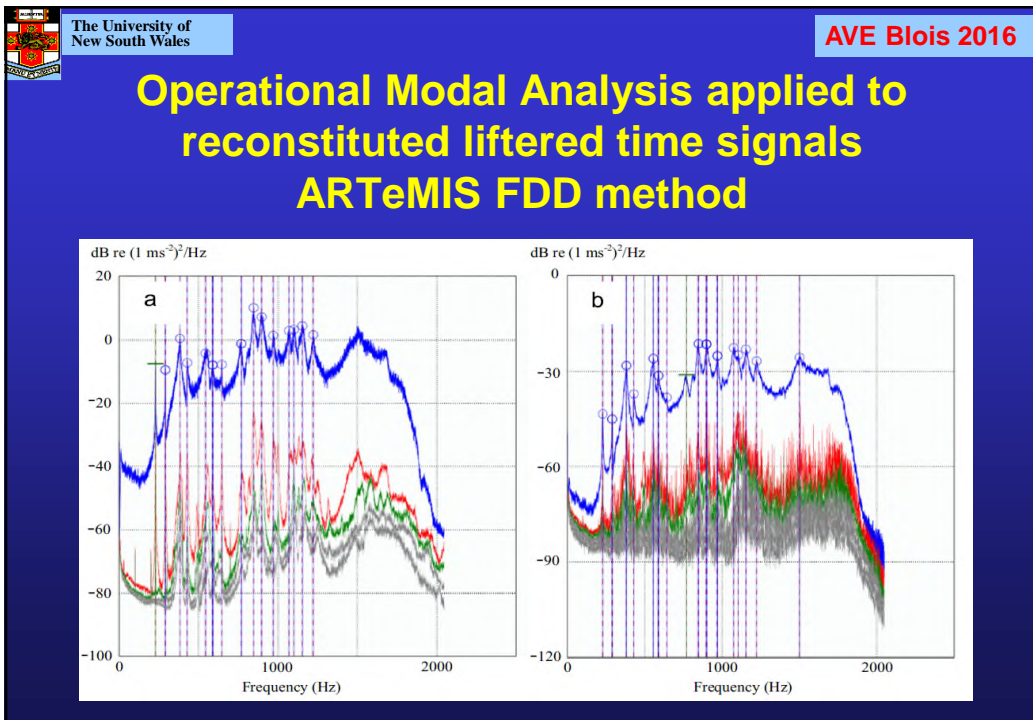


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## Exponential lifter result with $\pm 5\%$ speed variation







## CONCLUSION

- Le cepstre révèle la structure périodique dans un spectre (log), y compris les harmoniques, les bandes latérales et les échos
- Les effets de la fonction d'excitation et de transfert sont additifs dans le cepstre pour une seule entrée - application à l'analyse modale (opérationnelle)
- Signaux temporels peuvent être modifiés (par exemple suppression des échos) en utilisant le cepstre complexe, mais seulement lorsque la phase peut être déroulée
- Les signaux de réponse stationnaires peuvent maintenant être édités en utilisant le cepstre réel au lieu du cepstre complexe
- De nouvelles méthodes de lifrage cepstral suppriment les composants parasites, tels que des fréquences discrètes - utiles dans l'AMO
- Un « liftre » exponentiel supprime en aveugle de nombreuses perturbations, tout en se concentrant les informations modales à basse quéfrencce
- Pour le diagnostic des engrenages, les effets modaux devraient être supprimés pour la vitesse variable, le contraire pour les roulements