OPERATIONAL MODAL ANALYSIS DURING CYCLING

Analyse modale opérationnelle en cyclisme

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Abstract

The rider's comfort is an essential parameter in the design of a bike. A cyclist can often spend more than six hours on their bikes during a race and accumulate over 3000 km during competition. However, recent studies demonstrated that exposure to vibration for only 7 minutes on a cobblestone road can cause detrimental effects. The aim of this work is to apply operational modal analysis (OMA) method to study the dynamic behaviour of a bike-rider system. The experiments are conducted in two stages: (i) a voluntary subject carried out 10 tests on a cobblestone road at a speed of 20 km/h to study the repeatability of the method, and (ii) two volunteers with different mass performed a test on a cobblestone road at speeds of 10 and 20 km/h. For these two stages, the bike used has an aluminium frame and a carbon fork. A frequency domain technique is applied to extract the modal parameters of the bicycle structure. The modes are identified in the range of 5-50 Hz. It turns out that the OMA gives repeatable results for the modal frequencies (coefficient of variation CV = 4.2%), but not for the modal damping (CV = 75.3%) and mode shapes (modal vectors accuracy = 12%). It is shown that the speed and mass of subjects significantly influences the estimated modal frequency (relative deviation of 37.0% and 17.2% respectively)

Keywords: *Operational modal analysis, cycling, human vibration exposure.*

Résumé

Le confort du cycliste est l'un des paramètres essentiels dans la conception d'un vélo. Il peut rouler six heures lors d'une course et peut accumuler 3000km durant des compétitions. Cependant, des études récentes ont montré qu'une route pavée pouvait causer des effets délétères après 7 min d'exposition. Le but de ce travail est d'appliquer l'analyse modale opérationnelle (OMA) pour étudier le comportement dynamique du système vélo-cycliste. Les expérimentations se sont déroulées en deux étapes: (i) un sujet volontaire réalise 10 essais sur une route pavée à la vitesse de 20km/h pour tester la répétabilité de l'OMA, et (ii) deux volontaires de masses différentes réalisent un test sur une route pavée aux vitesses de 10 et 20km/h. Pour ces deux étapes, le vélo utilisé possède un cadre aluminium et une fourche carbone. Une technique fréquentielle est appliquée pour extraire les paramètres modaux de la structure. Les modes sont identifiés dans la plage 5-50Hz. Les résultats montrent que l'OMA donnent des résultats répétables pour les fréquences modales (coefficient de variation CV=4.2%), mais pas pour les amortissements (CV=75.2%) ni pour les déformées modales (Exactitude=12%). Egalement, il est montré que la vitesse et la masse du sujet influencent significativement les fréquences modales estimées (différence relative de 37.0% et 17.2% respectivement.

Mots clés : *Analyse modale, cyclisme, exposition humaine aux vibrations..*

1 Introduction

Human exposure to vibration can cause many health disorders [1]. The effects of exposure have been widely studied in the industrial world. They are estimated by calculating the vibration dose value for whole body [2] and hand-arm system [3]. From these standards, a threshold of daily stress limit is defined [4]. On the contrary, vibrations in sport have not been studied in depth despite the strong vibratory stress encountered in some disciplines. This paper is particularly interested in the case of cycling and motivated by the recent findings regarding the vibration exposure. Indeed, it was shown by Chiementin et al. [5] that the time exposure limit is 7 min for a cobblestone road, while the mean time spent on such pavements can be up to several hours during competition. Munera et al. [6] reported the effect of vibration on the performance and health of the cyclist. Pathological disorders developed are principally vascular [7, 8], but can also be neurological [9, 10].

The efforts of manufacturers and researches are focused on reducing the vibration dose absorbed by the cyclist. Munera et al. [11] developed a model of vibration transmission that predicts the vibration dose value in the hand-arm system. This study showed that an optimized geometry can reduce the vibration dose value by up to 50%. Lepine et al. [12] showed that the posture of the rider has a significant influence on the vibration dose value, in particular the position of the forearms and wrist angles. In other studies, Lepine et al. [13] proved the influence of different components of a bike on the vibration transmission. The authors showed that the fork and handlebar are the predominant vibrational factors for the hands and frame and wheels for the seat.

Moreover, a decrease in the vibratory dose values is possible thanks to an improved understanding of the dynamic behaviour of the bike. Richard et al. [14] showed that the use of modal analysis in laboratory conditions and field conditions is useful. Modal analysis allows for a better understanding of the natural behaviour of the bike and its rider through the natural frequencies, damping and mode shapes. These dynamic characteristics are determined through the measurement of the applied excitation force and the vibrational response of the structure [15]. The study by Champoux et al. [16] revealed that the identification method has little influence on the modal analysis results contrary to the presence of a rider within the frequency band of 0-300 Hz. This research was performed for a limited number of external conditions to the bike. The use of operational modal analysis depends on the quality of solicitation and on the specified boundary conditions.

This paper proposes (i) to validate the repeatability of operational modal analysis, and (ii) to evaluate the effects of cycling speed and mass of the subject on the operational modal analysis (OMA) results. This paper is structured around five parts. Section 2 explains the theory of operational modal analysis, in particular the useful assumptions for its application. Section 3 describes the experimental approach for identifying the modal characteristics of the bike for two speed conditions of 10 and 20 km/h and two subjects with a mass of 65 and 100 kg. Section 4 presents the experimental results and finally are discussed in Section 5.

2 Poly-Least Squares Complex Frequency Domain method

OMA represents a set of modal identification methods based only on the measurement of the system's response. Among the most efficient methods to extract the modal parameters and separate the closest modes, it can be highlighted the Poly-Least Squares Complex Frequency Domain (p-LSCF) method, [17]. It is based on the analysis of the matrix of power spectral densities of the output measurements, S_{yy}, (eq. 1).

$$S_{yy}(\omega) = H(\omega).S_{xx}(\omega).H(\omega)^H \quad \text{(eq. 1)}$$

$H(\omega)$ is composed by the frequency output function, $\bullet^H$ is the complex conjugate transpose of a matrix (Hermitian), and S_{xx} is the matrix of power spectral densities of the inputs.

In this case, we admit the hypothesis of white sources. Thus, the half spectral matrix $S^+_{yy}(\omega)$ could be decomposed as showed in (eq. 2).

$$S^+_{yy}(\omega) = \sum_{k=1}^{n} \Big(\frac{\phi_k.g_k^T}{i\omega - \lambda_k} + \frac{\phi_k^*.g_k^H}{i\omega - \lambda_k^*}\Big) \quad \text{(eq. 2)}$$

$\bullet^T$ is the transpose of a matrix, $\bullet^*$ is the complex conjugate of a matrix, $\bullet^H$ is the complex conjugate transpose of a matrix (Hermitian), ϕ_k is the mode shape k, g_k is the modal participation factor of mode k and λ_k are the structure poles.

The p-LSCF method requires the definition of a parametric model [18]. This paper uses the model right matrix fraction description (RMFD), presented in (eq. 3).

$$S^+_{yy}(\omega) = B(\omega)A(\omega)^{-1} \quad \text{(eq. 3)}$$

A and B are the denominator and numerator of the polynomial matrix defining the RMFD model respectively, (eq. 4).

$$B(\omega) = \sum_{j=0}^{p} \beta_j e^{i\omega\delta t j} \quad A(\omega) = \sum_{j=0}^{p} \alpha_j e^{i\omega\delta t j} \quad \text{(eq. 4)}$$

δt is the time of sampling. Thus, the purpose of the LSCF method is to identify all the factors α_j and β_j of the unknown model RMFD, minimizing the error using a nonlinear least-squares equation. This minimization is performed between the matrix of power spectral densities of the output measurements $S^+_{yy}(\omega)$ and RMFD model, (eq. 5), [17].

$$\epsilon(\omega_i) = B(\omega_i)A^{-1}(\omega_i) - S^+_{yy}(\omega_i) \quad \text{(eq. 5)}$$

From the α_j determined, the eigenvalues λ_k and k eigenvectors ϕ_k associated can be derived from the solution of (eq. 6), [19].

$$A(\omega) = 0 \quad \text{(eq. 6)}$$

The modes shapes are obtained by linear least-squares problem and are based on (eq. 2). The equation (eq. 7) is obtained by integrating two residual terms to approximate the effects outside the band, [20].

$$\widetilde{S^+}_{yy}(\omega) = \sum_i \left(\frac{\phi_i g_i^T}{j\omega - \lambda_i} + \frac{\phi_i^* g_i^H}{j\omega - \lambda_i^*} + \frac{LR}{j\omega} + j\omega UR\right) \quad \text{(eq. 7)}$$

LR and UR are the upper and lower residues and illustrate the influence of the modes out of the frequency band. g_i is the participation of mode i.

3 Material and methods

3.1 Subjects

Two healthy male cyclists (subject #1: 175 cm, 65 kg; subject #2: 190 cm, 100 kg), without apparent physical contraindications to whole body vibration exposure, voluntarily participated in this study. The exclusion criteria for the test subjects were implants, recent fractures, musculoskeletal disorders, kidney stones, diabetes, cardio-vascular disease, gallstones or epilepsy. The subjects gave a signed informed consent, and the study was approved by the local Ethics Committee.

Tab. 1. Bike characteristics.

Frame:	Aluminium 6061 T6 double butted
Seatstays:	High modulus carbon
Fork:	Carbon
Wheels:	B'twin forme plate 32
Air tube:	Hutchinson (pressure: 6 bars)
Mass:	9.8 kg

3.2 Material

The bike used in the study is detailed in Table 1.

In order to obtain characteristic signals of the frame, an OROS OR 35 (OROS, Grenoble, France) data acquisition system was used, Figure 1. It has eight universal inputs and two triggers/generators outputs. The data acquisition was controlled using the NVGATE software (OROS, Grenoble, France) loaded on a tablet computer, shown in Figure 1. The system is able to be carried during testing in a specially designed backpack. In all, the equipment has a mass of 3.6 kg.

The tablet computer and data collector are connected via an Ethernet crossover cable. A 12 V battery pack is used to power the system. The data acquisition is triggered by a push button positioned on the handlebar. The push button is connected to a system output channel generating a DC voltage and an input which is parametrized as a trigger. The action on the push button by the cyclist generates a rising edge which triggers the acquisition. A delay of the acquisition of one second is set, allowing the rider to correctly place their hands on the handlebar.

Fig. 1. (Left) Backpack with the data collector and the tablet computer. (Right) Cobblestone road.

The bike is equipped with two accelerometers (Brüel and Kjær Deltatron 4524B, frequency range: 0.25-3000 Hz, sensitivity: 10 mVg^{-1}). The first was positioned on the frame near the handlebars. Its position was fixed and considered as a reference in order to generate the deformed shape. The second sensor sweeps the frame on 14 locations, Figure 2. Cables between the accelerometers and the data collector were tied or glued onto the frame with adhesive tape in order to avoid any inconvenience to the rider.

3.3 Experimental procedure

The study is divided into two protocols. The first protocol is dedicated to testing the repeatability of the results of the OMA. The second protocol shows the influence of the cycling speed and the mass of the cyclist on the results. All tests were conducted on the same

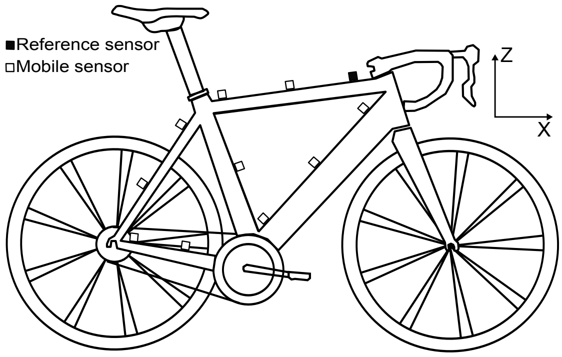

Fig. 2. Accelerometers position on the bicycle frame.

section of cobblestone roads in Reims (France), shown in Figure 1. This section is flat. Both protocols were carried out over several days in order to prevent rider fatigue. All tests were undertaken at a temperature of 22±3 °C and a hydrometry of 57±1%. Cycling speeds were controlled using a speedometer (Sigma Sport BC 16.12sts, Germany).

Protocol 1 is aimed at determining the modal characteristics from 10 tests. Each test consists of 14 individual measurements due to the different mounting locations of the accelerometer, thus this protocol requires 140 measurements in total. Each measurement lasts 10 seconds, with a recovery time of 2 minutes between each test. The posture of the subject is visually inspected. The arms are positioned on the top of the handlebar. Great attention was paid to the position of the hands, and to respect the angles. The speed was 20.2±0.2 km/h.

For the protocol 2, each subject undertakes two tests; one at a speed of 10.0 km/h and a second at 20.0 km/h. Between the two tests, the participants are allowed 2 minutes of recovery time. The two tests are repeated for all the positions of the mobile sensor. Each test requires 28 measurements for each test subject.

3.4 Data analysis

For both protocols, modal characteristics are calculated by the p-LSCF method described in section 2. Obtaining the mode shapes requires the model to be constructed with a simple geometry. Thus a bicycle frame is made from different nodes similar to the measurement points. This geometry is made using the MODAL software (OROS, Grenoble, France), shown in Figure 3.

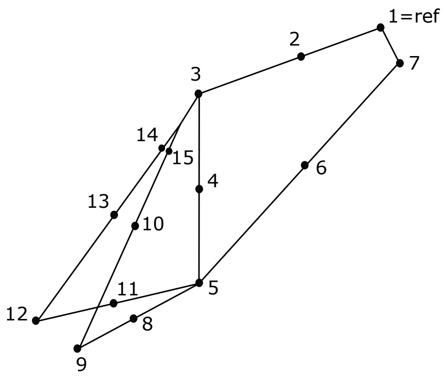

Fig. 3. Geometry of the bike frame in the Modal analysis software.

The vibration signals are recorded with a sampling frequency of 2560 Hz. Then, they were decimated to reduce the frequency range to 0-250 Hz. Preliminary measurements show that for frequencies over 200 Hz the spectral signal is similar to noise. The Fourier

Tab. 2. Identification results on the 0-100 Hz band. The eigen frequencies and damping ratio are expressed as mean ± standard deviation (coefficient of variation). Score is the number of tests where Mode i is identified.

# Mode	Score $/10$	Frequency [Hz] m ±std(CV)	Damping ratio % m ±std (CV)
1	10	9.03±0.46 (0.05)	6.77±1.62 (0.24)
2	9	20.73±0.86 (0.04)	1.30±0.66 (0.50)
3	1	26.90± - (-)	0.60± - (-)
4	8	32.33±0.76 (0.02)	1.23±1.80 (1.46)
5	10	49.00±3.12 (0.06)	1.43±1.16 (0.81)

transform is computed for 1024 samples and a rectangular sliding window with 50% overlap. The modal identification procedure is performed thanks to a stability diagram.

Protocol 1 relates to the repeatability. The modal characteristics of the bike-rider system, specifically the modal frequencies, damping and shapes are determined. The estimates of the frequencies and damping are presented as mean±standard deviation (coefficient of variation). The coefficient of variation is calculated as the ratio between the standard deviation and the mean. Mode shapes are compared by using the MAC matrix (Modal Assurance Criterion). The MAC matrix allows the comparison of two sets of eigenmodes $\{\phi_i^1\}$ and $\{\phi_j^1\}$, given in (eq. 8). Ewins et al. [21] defines that the eigenmodes are not correlated for a value less than 0.20 and are well correlated above 0.80. Each element of this matrix is presented as mean ± standard deviation (coefficient of variation).

$$(MAC)_{i,j} = \frac{\left(\{\phi_i^1\}^T\{\phi_j^2\}^*\right)^2}{\left\|\{\phi_i^1\}\right\|^2 \left\|\{\phi_j^2\}\right\|^2} \; i = 1, ..., n; j = 1, ..., p \qquad \text{(eq. 8)}$$

Protocol 2 is related to the effects of the parameters. The modal characteristics of the bicycle are determined: modal frequencies, damping and shapes, as indicated above. The effects of the mass and speed are evaluated by the averaged absolute values of the relative deviations (expressed as a percentage).

4 Results

4.1 Repeatability

The repeatability of the results are listed in the Table 2. The modal density in the 0-100 Hz frequency band is relatively low. Five modes are identified, and are concentrated within 0-50 Hz. The 1st, 2nd, 4th, and 5th modes are identified in over 80% of the experiments. Mode 3 was identified in only one test.

The eigen frequencies of modes 1, 2, 4, and 5 have a coefficient of variation of 6%. The variation in the 3rd mode cannot be determined because it was only able to be identified in one test. Observation of the stability diagram reveals that the identified modes have a frequency of pole invariance under 1% tolerance.

Damping ratio for modes 2, 4, 5 have similar values, approximately 1.32%. The damping of the 1st mode is 5.5 times higher, while mode 3 is 2 times lower. It was found that the variation coefficients have values ranging from 24% for mode 1 up to 146% for mode 4. For the same reasons than for the eigen frequencies, the coefficient of variation for mode

Tab. 3. Mean MAC matrix

# Mode	1	2	3	4	5
1	0.56±0.08	0.19±0.06	0.34±0.14	0.08±0.03	0.32±0.14
2	0.21±0.10	0.60±0.08	0.21±0.20	0.63±0.21	0.23±0.12
3	0.35±0.13	0.22±0.10	0.68±-	0.22±0.06	0.29±0.15
4	0.05±0.01	0.26±0.12	0.21±0.17	0.75±0.11	0.03±0.02
5	0.19±0.05	0.25±0.08	0.32±0.15	0.65±0.21	0.21±0.05

Tab. 4. Results of the modal identification. Frequency [Hz] (damping ratio %).

	65 kg		100 kg	
# Mode	10 km/h	20 km/h	10 km/h	20 km/h
1	8.3 (3.8)	9.9 (2.0)	8.5 (5.8)	5.4 (0.1)
2	20.2 (0.3)	20.7 (4.0)	24.4 (0.2)	18.9 (2.2)
3	- (-)	- (-)	- (-)	26.9 (0.6)
4	30.9 (1.8)	30.2 (3.9)	29.3 (3.1)	34.3 (3.2)
5	51.5 (2.0)	48.5 (4.2)	48.5 (4.2)	55.4 (0.7)

3 cannot be calculated. The stability diagram shows that the identified modes present a damping and an eigen vector unstable. The correlations of the mode shapes extracted from the analysis are shown by the MAC matrix, Table 3. The diagonal of this table shows a maximum correlation of 0.75 while the values should exceed 0.80. Values outside of the diagonal have values higher than 0.20 which do not show the independence between these modes except for modes 1 and 3, modes 3 and 5, and modes 5 and 1. The theoretical MAC matrix is satisfactory in 12% of cases.

4.2 Effect of cycling speed and mass

The results of the modal identification for the two speeds and two masses are listed in Table 4.

For all these conditions, 4 modes were identified: modes 1, 2, 4, and 5. The frequencies are identified on respective statistical ranges of 6.5, 5.5, 6.1 and 6.9 Hz. The damping are identified on respective statistical ranges 5.8, 3.8, 2.1, 3.5%. The condition 20 km/h - 100 kg identifies one additional mode to 26.90 Hz with a 0.6% damping. The cycling speed was found to greatly influence the estimated eigen values. In the general case, a speed increase of 5.0% decreases the frequency values and increases damping of 11.0%. For the experiment with a rider mass of 65 kg, the frequencies and damping increase by 2.9 and 63.6% respectively. For 100 kg rider, the frequencies and damping decreased by 12.9 and 41.6% respectively. The maximum variation in frequency and damping is achieved for 100 kg provided in mode 1, the value reached -55.0% and -193.2% respectively. Figure 4 details the effect of speed on each mode. The increase in mass decreases the frequency

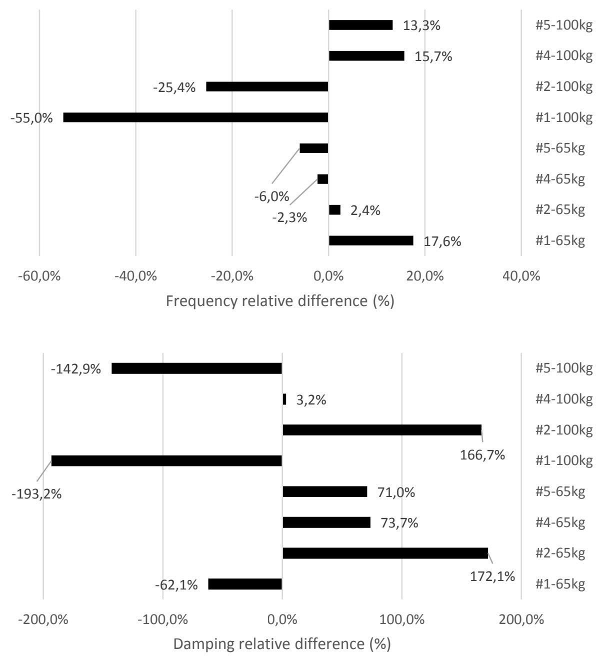

Fig. 4. Cycling speed effect. Relative difference (%) for frequencies (top) and for damping (bottom)

and damping by 2.6% and 34.5% respectively. However the trends are not similar at both speeds evaluated. The speed of 10 km/h increased the estimated frequencies and damping by 5.3% and 31.4%, respectively. For the 20 km/h test, a decrease in the frequencies and damping by 10.5 and 100.4% was found, respectively. The greatest variation is noted for mode 1 at 20 km/h; the value is −58.8% for the frequency and −181.0% for damping. Figure 5 describes the effect of the mass on each mode.

5 Discussions

With the proposed protocol, the implementation of OMA on the bike-rider system enables for repeatable identification of the modal parameters of the system. Of all the modes, only 4 modes were identified in more than 80% of the tests. The estimated natural frequencies were found to be identical for the 4 identified modes with a coefficient of variation under 5%. However, the damping ratio was found to have a very high coefficient of variation (CV = 146%). The MACs matrix is not satisfactory, with an overall accuracy of 12%.

The cycling speed has a significant influence on the measured results. An increase of 10 km/h generates variations in frequency and damping ranging from −55.0% to 17.6% and −193.2% to 166.7%, respectively. This fact proves that the rider creates a different behaviour according to the cycling speed. The activation of muscle groups depends largely on the nature of the excitation. Indeed, Munera et al. [22] have shown that the muscles had greater contractions for a high level of vibration. Thus, the rigidity of the bike-rider system could be modified.

The speed increase was found to involve two opposite behaviours of the natural frequencies, depending on the mass of the subject. The increase in speed reduces the two first eigen frequencies and increases the two second eigen frequencies for the 100 kg subject, while the changes are reversed for the 65 kg subject.

The mass of the rider also has an effect on the eigen frequencies. It is shown that an increase in mass will result in a decrease of the first two natural frequencies and an increase in the 4th and 5th modes. Given that Crequy [23] shows that increasing the rider's mass decreased the vibration dose value, it is possible to correlate this dose value with the modes 4 and 5. These modes are related to torsion of the fork and plane movement of the frame.

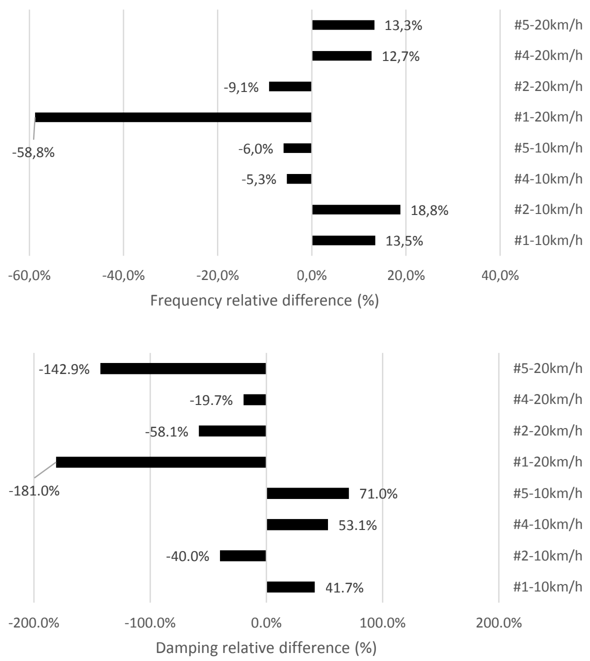

Fig. 5. Mass effect. Relative difference (%) for frequencies (top) and for damping (bottom)

In general, the natural frequencies are in the 5-50 Hz range. The ISO2631 and ISO5349 standards, related to the body and the hand arm system respectively, are based on a frequency weighting which represents the probability of risks. The weighting given by the ISO2631 standard is at a maximum within the frequency band of 4-10 Hz, and for frequencies over the band 2-20 Hz the weighting is at least halved. For the ISO5349 standard, the weighting is at a maximum for the range 8-16 Hz, while it is reduced by more than half over the range 5-35 Hz. In both cases, the bicycle modes are in the band where the probability of risks is higher, and more particularly within the first two modes identified in this study. With this remark, the effect of the first two modes must get under control in order to reduce the harmful risks. Based on the deformed shapes identified in the study, the fork is involved in the resonance phenomenon. This was confirmed in a study by Lepine et al. [13]. Thanks to a factorial design test, this work shows that the fork and the wheel are predominant in the transmission of vibrations.

6 Conclusion

Understanding the dynamic behaviour of the bike-rider system can be a major asset to identify the mechanical stresses and vibration experienced by the bicycle and cyclist. This paper implemented OMA on vibration measured on a bicycle in the field. The analysis of the method found that the identification of frequencies was repeatable despite to the poorly estimated damping and deformed shapes. Next, OMA was applied under conditions with varying rider mass and cycling speed. It was shown that these parameters have a significant influence on the results. The cycling speed and mass of the subjects significantly influenced the results in the estimated frequency (relative deviation = 37.0% and 17.2% respectively). From the weightings outlined in the standards, the first two modes of the studied bike seems to be monitored. It is important that, during the design of a bike, the 5-20 Hz frequency range must be considered.

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